Lesson: Dimensional Analysis (Lesson #1 is vector resolution not dimensional analysis)
Grade Level 11-12, Physics

Lesson Overview:
This lesson will be our first physics lesson chapter one section one and define what physics is. Physics is the branch of science that involves the study of the physical world; matter and energy and how they are related. Physics uses math as it language. The last statement is one of the most powerful physics statement made. We must have a firm understand of dimensional analysis before we can continue on our quest for physical knowledge. While taking ED5661 one of the most important concepts was doing conversions from degrees minutes and seconds and also statute miles and regular miles and the understanding on knots and the relationship to miles per hour.

Objectives:
Use the metric system.
Evaluate answers using dimension analysis.
Perform arithmetic operations using feet yard miles meters and statute miles.
Perform conversions between degrees minutes and seconds.

Vocabulary
Physics
Dimensional analysis
Significant digits
Scientific method
Scientific laws
Scientific theory
Nautical mile
Statute mile
Knot
What we will learn:

You will apply accuracy and precision when measuring
You will display and evaluate data.
You will learn basic conversion rates.

Why is it important?

The measurement and mathematical tools present here will help you analyze data and make predictions.

Key mathematical terms constants and conversions.

<table>
<thead>
<tr>
<th>Factor Name</th>
<th>Symbol</th>
<th>Factor Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-24}$</td>
<td>yotta</td>
<td>$10^{-1}$</td>
<td>deci</td>
</tr>
<tr>
<td>$10^{-21}$</td>
<td>zetta</td>
<td>$10^{-2}$</td>
<td>centi</td>
</tr>
<tr>
<td>$10^{-18}$</td>
<td>exa</td>
<td>$10^{-3}$</td>
<td>milli</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>peta</td>
<td>$10^{-6}$</td>
<td>micro</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>tera</td>
<td>$10^{-9}$</td>
<td>nano</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>giga</td>
<td>$10^{-12}$</td>
<td>pico</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>mega</td>
<td>$10^{-15}$</td>
<td>femto</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>kilo</td>
<td>$10^{-18}$</td>
<td>atto</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>hecto</td>
<td>$10^{-21}$</td>
<td>zepto</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td>deka</td>
<td>$10^{-24}$</td>
<td>yocto</td>
</tr>
</tbody>
</table>

CONVERSION TABLE FOR NAUTICAL AND STATUTE MILES

1 nautical mile = 6,076.11549 . . . feet
1 statute mile = 5,280 feet
In a full circle there are 360 degrees.

Each degree is split up into 60 parts, each part being 1/60 of a degree. These parts are called minutes.

Each minute is split up into 60 parts, each part being 1/60 of a minute. These parts are called seconds.

<table>
<thead>
<tr>
<th>English &amp; Metric Conversion Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch =</td>
</tr>
<tr>
<td>1 feet =</td>
</tr>
<tr>
<td>1 yard =</td>
</tr>
<tr>
<td>1 rod / perch =</td>
</tr>
<tr>
<td>1 chain =</td>
</tr>
<tr>
<td>1 furlong =</td>
</tr>
<tr>
<td>1 mile =</td>
</tr>
<tr>
<td>1 mile =</td>
</tr>
<tr>
<td>1 hand =</td>
</tr>
<tr>
<td>1 mil =</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nautical Conversion Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 fathom =</td>
</tr>
<tr>
<td>1 cable length =</td>
</tr>
<tr>
<td>1 nautical mile =</td>
</tr>
<tr>
<td>1 league =</td>
</tr>
</tbody>
</table>

Materials needed:

Paper and pencil
Calculators
Books:


Physics (Principals and Problems) Glencoe Science 2009

Standards:

(Even thought dimensional analysis is not specifically mention in theses state standards it is required as prior knowledge and has to be taught in my physics class) many if not all of the following standard need this fundamental concept to continue.

**Standard P1: INQUIRY, Reflection, And social implications**

*Students will understand the nature of science and demonstrate an ability to practice scientific reasoning by applying it to the design, execution, and evaluation of scientific investigations. Students will demonstrate their understanding that scientific knowledge is gathered through various forms of direct and indirect observations and the testing of this information by methods including, but not limited to, experimentation. They will be able to distinguish between types of scientific knowledge (e.g., hypotheses, laws, theories) and become aware of areas of active research in contrast to conclusions that are part of established scientific consensus. They will use their scientific knowledge to assess the costs, risks, and benefits of technological systems as they make personal choices and participate in public policy decisions. These insights will help them analyze the role science plays in society, technology, and potential career opportunities.*

**P1.1 Scientific Inquiry**

Science is a way of understanding nature. Scientific research may begin by generating new scientific questions that can be answered through replicable scientific investigations that are logically developed and conducted systematically. Scientific conclusions and explanations result from careful analysis of empirical evidence and the use of logical reasoning. Some questions in science are addressed through indirect rather than direct observation, evaluating the consistency of new evidence with results predicted by models of natural processes. Results from investigations are communicated in reports that are scrutinized through a peer review process.

**P1.1A** Generate new questions that can be investigated in the laboratory or field.

**P1.1B** Evaluate the uncertainties or validity of scientific conclusions using an understanding of sources of measurement error, the challenges of controlling variables, accuracy of data analysis, logic of argument, logic of experimental design, and/or the dependence on underlying assumptions.
P1.1C Conduct scientific investigations using appropriate tools and techniques (e.g., selecting an instrument that measures the desired quantity–length, volume, weight, time interval, temperature–with the appropriate level of precision).

P1.1D Identify patterns in data and relate them to theoretical models.

P1.1E Describe a reason for a given conclusion using evidence from an investigation.

P1.1f Predict what would happen if the variables, methods, or timing of an investigation were changed.

P1.1g Based on empirical evidence, explain and critique the reasoning used to draw a scientific conclusion or explanation.

P1.1h Design and conduct a systematic scientific investigation that tests a hypothesis. Draw conclusions from data presented in charts or tables.

**Classroom example #1 (Basic conversions)**

**Lecture 20 minutes.**

Problem #1

I know to people who practice math regularly, this will be common sense, but I haven't been in math for a year and I'm a little out of it. I'm in an oceanography class and 1 nautical mile=1.15 statute miles. If I have 617 nautical miles, how many statute miles is that?

\[ 1\text{nm} = 1.15\text{sm} \]
\[ 617\text{ nm} = x\text{ sm} \]

Just multiply 617 times 1.15

709.55

Problem #2 (some basic knowledge of nautical mile)

1. How many feet are in a nautical mile? Meters are in a nautical mile?
2. Convert the deepest point in the ocean, the Marianas Trench at a depth of 35,802 feet to meters.
3. The largest and fastest marine fish is the blue fin tuna weighing 1,500 pounds and swims up to 55 miles per hour. How fast is this in knots?
4. Convert 20,000 miles to statute miles. Is it possible for such depths to occur in the ocean?
Conversion Table:
1 fathom = 6 feet 100 fathoms = 1 cable length 10 cables length = 1 nautical mile 1 nautical mile = 1.151 statute miles (length of a minute of longitude at equator) 3 nautical miles = 1 league 1 knot = 1 nautical mile per hour = 1.151 statute miles per hour 10 chains = 1 furlong = 201.17 meters 60 nautical miles = 1 degree of a great circle of earth (latitude) 1 statute mile = 5280 feet 1 nautical mile = 6076.115 feet

Note: Statute miles is what we know as miles and is a measurement of distance over land. Nautical miles are a measurement over water.

Origin of Nautical Terms

Fathom - Sailors use to throw a line into the water, wait until it hit the bottom, pull it back up, while measuring the length of the line from finger tip to finger tip. The arm span of an average sailor was 6 feet and called a fathom.

Knot - Lines use to be thrown over the sides of ships to determine speed. Each line was divided into 47 ft. 3 in. sections and was called knots. The line was allowed to run over the ship's side while a 28-second glass was emptying itself. The length of the knot was derived from the proportion that one hour (3600 sec) is to 28 seconds as one mile (6076.115 ft.) is to the length of one knot (47 ft. 3 in.)

Classroom example #2

Problem #1

If you are going 50 miles per hour, how many feet per second are you traveling?

If you were to do this one on the blackboard, it might look something like this:
You want your answer to be in feet per second. You are given 50 miles per hour. Notice that both are in distance per time. Normally you can use any value given by the problem as your starting factor. One thing you know, then, is given. The other things you just know or have to look up in a conversion table. Although every conversion factor can be written two ways, you really only need to write each one way. That's because you know you can always just flip it over and then use it. If you have written 60 min/1 hr, then to solve this problem you would just flip the 60 min/hour factor over. With practice you won't even need to write down what you know, you'll just pull it out of your head and write down the last part, do the math, and get the right answer.

Problem #2

You have come down with a bad case of the geebies, but fortunately your grandmother has a sure cure. She gives you an eyedropper bottle labeled:

Take 1 drop per 15 lb of body weight per dose four times a day until the geebies are gone. Contains gr 8 heebie bark per dr 100 solvent. 60 drops=1 tsp.

You weigh 128 lb, and the 4-oz bottle is half-full. You test the eyedropper and find there are actually 64 drops in a teaspoon. You are going on a three-week trip and are deeply concerned that you might run out of granny's geebie tonic. Do you need to see her before leaving to get a refill?

Try working this one out before reading further.

First, what do you want to know? You want to know how long the bottle will last. You could figure out days/bottle or weeks/bottle and see if the bottle will last longer than 3 weeks or 21 days. So you write down "days/bottle" as the units you want in your answer.
What do you know to start off with that you might need to know? You write down the following:

<table>
<thead>
<tr>
<th>want:</th>
<th>days</th>
</tr>
</thead>
<tbody>
<tr>
<td>know:</td>
<td>bottle</td>
</tr>
<tr>
<td></td>
<td>1 day 4 oz 1 bottle 2 half-bottle 2 oz tonic</td>
</tr>
<tr>
<td></td>
<td>64 drops 3 tsp 2 tbs 1 oz 1 drop 15 lb x dose</td>
</tr>
<tr>
<td></td>
<td>128 lb</td>
</tr>
</tbody>
</table>

You realize that if a 4-oz bottle is half-full, then there is 2 oz of tonic in it, but you could figure it out dimensionally if you wanted to:

\[
\frac{4 \text{ oz}}{1 \text{ bottle}} \times \frac{1 \text{ bottle}}{2 \text{ half-bottle}} = \frac{2 \text{ oz}}{\text{half-bottle}}
\]

You would then end up with "days/half-bottle" in your answer, but it's easier to just go with 2 oz/bottle as you're given.

What should you use as a starting factor? You pick 128 lb because it's something you're given and it seems lonely. You set the problem up:

<table>
<thead>
<tr>
<th>128 lb</th>
<th>1 drop</th>
<th>4 doses</th>
<th>1 tsp</th>
<th>1 day</th>
<th>64 drops</th>
<th>3 tsp</th>
<th>2 tbs</th>
<th>1 oz</th>
<th>1 drop</th>
<th>15 lb</th>
<th>dose</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 lb</td>
<td>drop</td>
<td>1 dose</td>
<td>1 day</td>
<td>64 drops</td>
<td>3 tsp</td>
<td>2 tbs</td>
<td>2 oz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Houston, we have a problem. You ended up with units reversed from what you wanted. You figured out how much of the bottle you would use in one day. What to do? You could hit the 1/x button on your calculator if it had one, or invert the answer by dividing 1 by 0.044, or start over with 128 lb on the bottom. What? Can you do that? Sure you can. You could even put 128 lb on the end and on the bottom, or put it in the middle somewhere. You decide to start over, this time picking a starting factor that already has "day" or "bottle" in the right place:

\[
\frac{2 \text{ oz}}{1 \text{ oz}} \times \frac{2 \text{ tbs}}{1 \text{ tsp}} \times \frac{64 \text{ drops}}{1 \text{ drop}} \times \frac{15 \text{ lb}}{1 \text{ dose}} \times \frac{1 \text{ day}}{1 \text{ bottle}} = 22.5 \text{ days/bottle}
\]

So, it looks like you'll have enough. At some point you need to know how many drops per dose you will need to take, so you figure it out:

\[
\frac{128 \text{ lb}}{15 \text{ lb/dose}} = 8.5333 = 9 \text{ drops/dose}
\]
As a practical matter, you can't take 8.533 drops per dose; you have to round off. At this point you realize that when you calculated 22.5 days/bottle, you were not figuring on 9 drops/dose. You decide to recalculate to see if rounding up to 9 makes a significant difference.

\[
\begin{array}{c|c|c|c|c|c|c}
2 \text{ oz} & 2 \text{ lbs} & 3 \text{ tsp} & 64 \text{ drops} & 1 \text{ dose} & 1 \text{ day} \\
\hline
\text{bottle} & 1 \text{ oz} & 1 \text{ lbs} & 1 \text{ tsp} & 9 \text{ drops} & 4 \text{ doses}
\end{array} = \frac{21.3 \text{ days}}{\text{bottle}}
\]

You note a small difference, but conclude that you have just enough geebie tonic. Concluding that you have enough, however, and having enough may not be the same thing. The story continues:

You leave on your trip and on the 19th day you run out of geebie juice. You didn't spill any, and no one took any. You sit in a stunned stupor trying to figure out where you went wrong in your calculations.

You finally realize there might not have been 2.0 oz of tonic in the bottle to begin with. A measurement like "half a bottle" should not inspire great certainty. You wish you had measured the amount and found that the bottle contained $2.0 \pm 0.05$ oz of tonic, but what you were given, more or less, was that you had $2 \pm 0.5$ oz of tonic. There could be anything from 1.5 to 2.5 oz in the bottle. Recalculating using the low and high values, you find you had enough tonic to last somewhere between 16 and 26 days. If you had figured out the correct answer of $21 \pm 5$ days the first time, you would have realized you had only slightly less than a 50/50 chance of running out, and would have gone to see Granny for a refill.

**Student Homework to be in class:**

Solve using the conversion factors that are listed in the table above.

a. Your cruise ship is leaving for a 610-mile adventure. How many nautical miles is this?

b. Later the ship is discovered at 38 fathoms deep under water. Convert this to meters.

c. Fortunately you survived! You are stranded on a deserted island that is located 12.5 miles north of the equator. How many nautical miles is this?

d. If you are rationed to 32 gills of fresh water a day. How many liters is this? 1 gill = 5 fish.